

Name: \_\_\_\_\_ Period: \_\_\_\_\_

# **Accel. Pre-Calculus**

## **Unit 2 Packet**

### **Trig Graphs**

2.01: Exploring the Sine & Cosine Function Graphs

Using the unit circle and a calculator, fill in the chart below with the **value of sine** at each angle.

Degrees	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Radians																	
Sine (exact)																	
Sine (decimal)																	

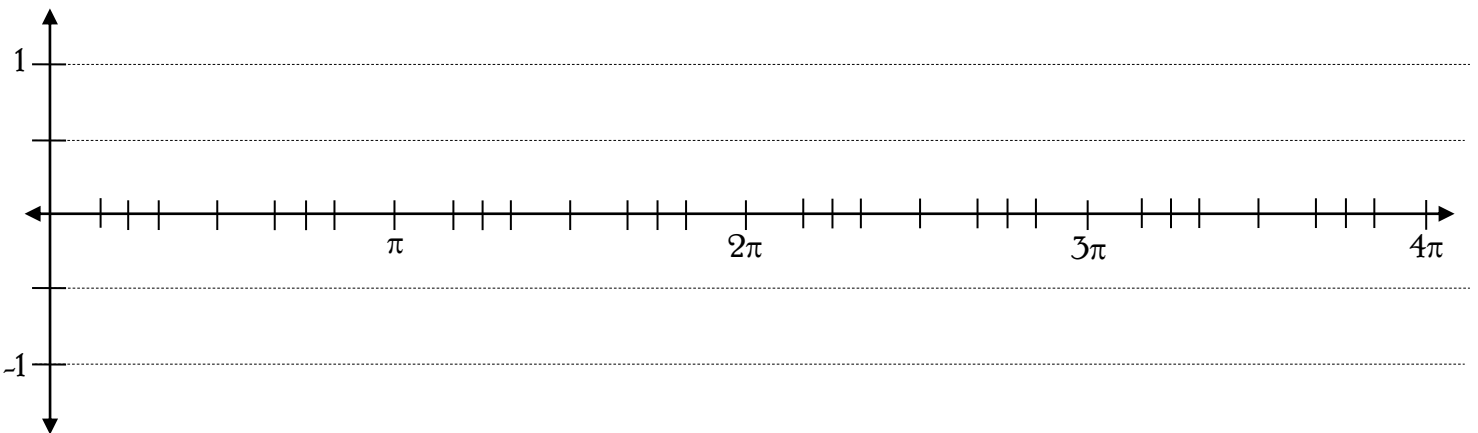
Graph  $y = \sin x$ .

This means the angle measure is plotted along the x-axis and the sine value is plotted along the y-axis.

Label all angles marked along the x-axis. Then, plot the points found in the table above:

$(x, y) = (\text{radian angle measure, sine value in decimal form})$

The x-axis is extended to  $4\pi$ . Plot points for angles through that second rotation around the unit circle.



- The period is \_\_\_\_\_. The **period** of a function is the distance required for it to complete one full cycle.
- The domain is \_\_\_\_\_. The **domain** of a function is the set of all possible input values.
- The range is \_\_\_\_\_. The **range** of a function is the set of all possible output values.
- The x-intercepts are \_\_\_\_\_. The **x-intercepts** of a function is the set of values where the graph crosses the x-axis.
- The y-intercept is \_\_\_\_\_. The **y-intercepts** of a function is the set of values where the graph crosses the y-axis.
- The maximum value is \_\_\_\_\_ and occurs when  $x =$  \_\_\_\_\_.
- The minimum value is \_\_\_\_\_ and occurs when  $x =$  \_\_\_\_\_.

Using the unit circle and a calculator, fill in the chart below with the **value of cosine** at each angle.

Degrees	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Radians																	
Cosine (exact)																	
Cosine (decimal)																	

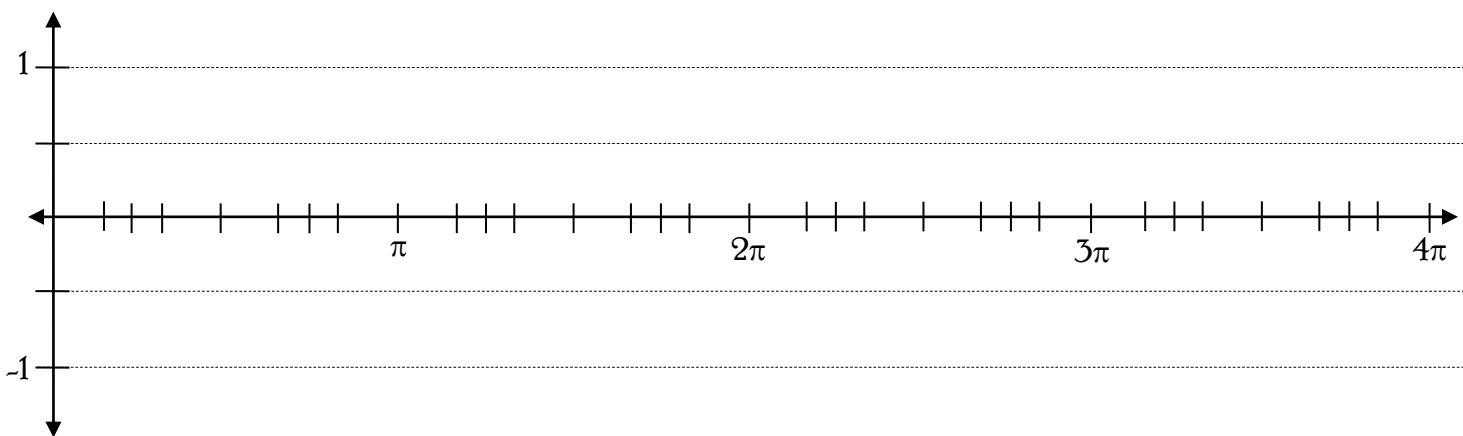
Graph  $y = \cos x$ .

This means the angle measure is plotted along the x-axis and the cosine value is plotted along the y-axis.

Label all angles marked along the x-axis. Then, plot the points found in the table above:

$$(x, y) = (\text{radian angle measure, cosine value in decimal form})$$

The x-axis is extended to  $4\pi$ . Plot points for angles through that second rotation around the unit circle.



1. The period is \_\_\_\_\_. The **period** of a function is the distance required for it to complete one full cycle.
2. The domain is \_\_\_\_\_. The **domain** of a function is the set of all possible input values.
3. The range is \_\_\_\_\_. The **range** of a function is the set of all possible output values.
4. The x-intercepts are \_\_\_\_\_. The **x-intercepts** of a function is the set of values where the graph crosses the x-axis.
5. The y-intercept is \_\_\_\_\_. The **y-intercepts** of a function is the set of values where the graph crosses the y-axis.
6. The maximum value is \_\_\_\_\_ and occurs when  $x =$  \_\_\_\_\_.
7. The minimum value is \_\_\_\_\_ and occurs when  $x =$  \_\_\_\_\_.

Accel Pre-Calculus  
 Graphing Sine and Cosine Functions  
 2.02 Amplitude and Period Notes

Name: \_\_\_\_\_

Date: \_\_\_\_\_

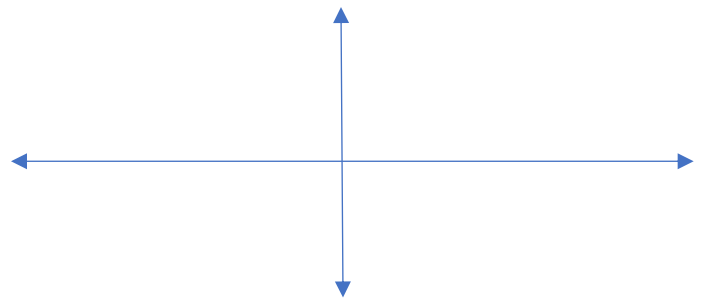
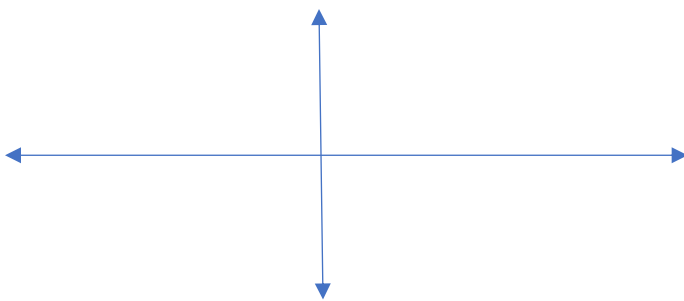
Remember the patterns and the shape of the graphs for the sine and cosine function:

$y = \sin\theta$

$\theta$					
$\sin\theta$					

$y = \cos\theta$

$\theta$					
$\cos\theta$					



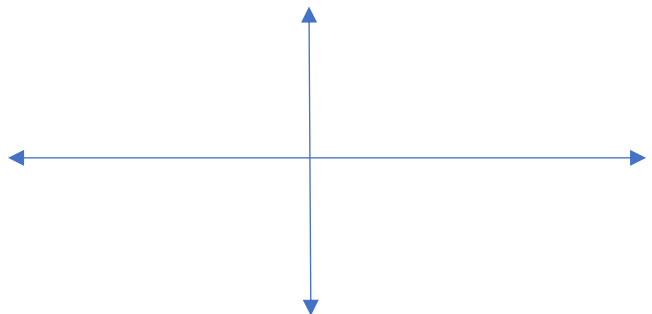
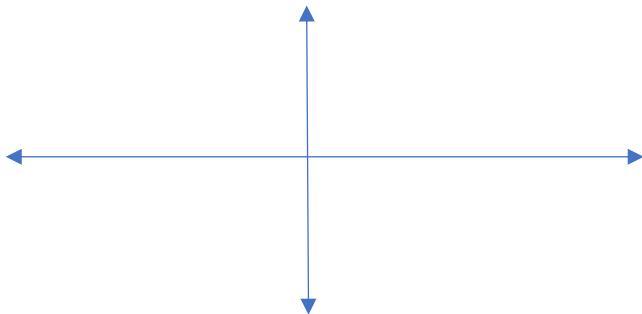
Given  $y = a\sin b\theta$  and  $y = a\cos b\theta$  we define the Amplitude as: \_\_\_\_\_

Examples:

- 1)  $y = \frac{1}{2}\sin\theta$   $a =$  \_\_\_\_\_ Amplitude = \_\_\_\_\_      2)  $y = -4\cos\theta$   $a =$  \_\_\_\_\_ Amplitude = \_\_\_\_\_

$\theta$					
$\sin\theta$					
$y = \frac{1}{2}\sin\theta$					

$\theta$					
$\cos\theta$					
$y = -4\cos\theta$					



We defined the period of the graph as: \_\_\_\_\_

Given  $y = asinb\theta$  and  $y = acosb\theta$  we use "b" to determine the change to the period of the graph:

Period ( $P$ ) =  $\frac{2\pi}{b}$ . Likewise, if you know the Period ( $P$ ) you can find  $b$ :  $b = \frac{2\pi}{P}$

Once we determine the period of the graph, we divide the period by 4 to determine the Interval in order to label the x-axis:  $Interval (I) = \frac{P}{4}$

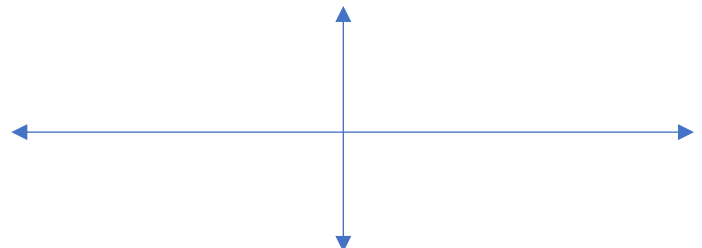
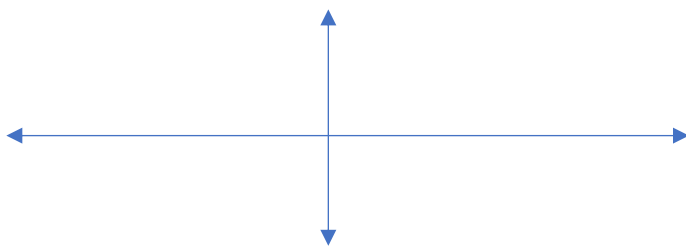
Examples:

3)  $y = sin2\theta$        $b =$  \_\_\_\_\_  
 Period = \_\_\_\_\_      Interval = \_\_\_\_\_

4)  $y = cos\frac{\theta}{2}$        $b =$  \_\_\_\_\_  
 Period = \_\_\_\_\_      Interval = \_\_\_\_\_

$\theta$					
$y = sin2\theta$					

$\theta$					
$y = cos\frac{\theta}{2}$					



Let's try to graph sine and cosine with changes to both the amplitude and the period.

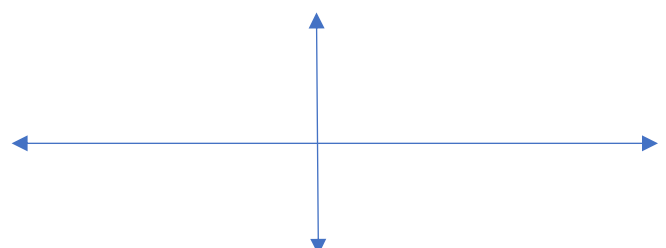
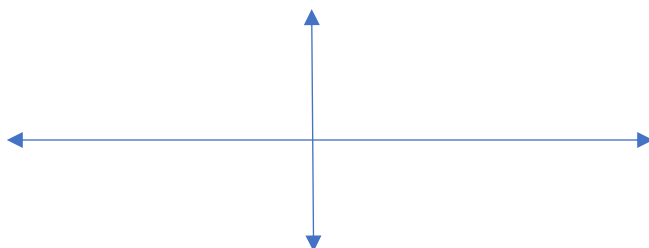
Examples:

5)  $y = -3sin4\theta$      $a =$  \_\_\_ Amplitude = \_\_\_    6)  $y = 2cos\frac{\theta}{3}$      $a =$  \_\_\_ Amplitude = \_\_\_

$b =$  \_\_\_\_\_ Period = \_\_\_\_\_ Interval = \_\_\_\_\_     $b =$  \_\_\_\_\_ Period = \_\_\_\_\_ Interval = \_\_\_\_\_

$\theta$					
$sin\theta$					
$y = -3sin4\theta$					

$\theta$					
$cos\theta$					
$y = 2cos\frac{\theta}{3}$					



2.02 Practice- Graphing Sin and Cos - Amplitude and Period      Date: \_\_\_\_\_

For each function, state the amplitude and period. Then label the axes appropriately and sketch the graph.

1)  $y = 5 \cos \theta$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

2)  $y = 3 \sin \theta$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

3)  $y = -\frac{1}{3} \cos \theta$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

4)  $y = -4 \sin \theta$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Write an equation of the function with the given properties:

5) A sine function with an amplitude of 0.4

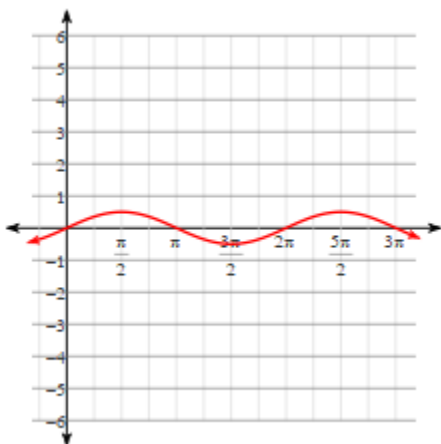
6) A cosine function with an amplitude of 7.5

7) A sine function with an amplitude of  $\frac{1}{4}$

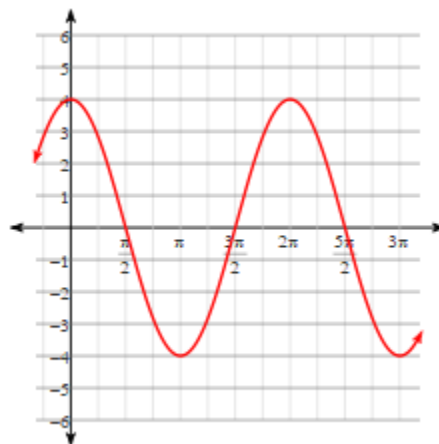
8) A cosine function with an amplitude of  $\frac{2}{5}$

Write an equation for each graph below:

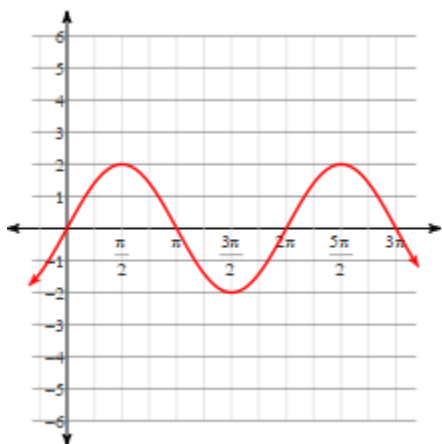
9)



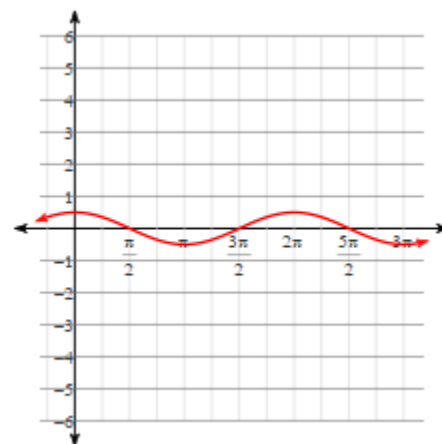
10)



11)



12)



**2.03 Practice: Graphing Sine & Cosine with Amplitude & Period** Date: \_\_\_\_\_

**State the amplitude and period for each function.**

1.  $y = -\frac{2}{5} \sin 9\theta$

2.  $y = \frac{2}{3} \cos \frac{3}{7} \theta$

3.  $y = -2.5 \cos \frac{\theta}{5}$

4.  $y = \frac{1}{3} \sin \frac{\theta}{3}$

**Write an equation of the sine function with each amplitude and period.**

5. amplitude = 3, period =  $\frac{\pi}{6}$

6. amplitude = 6, period =  $3\pi$

7. amplitude = 2, period =  $10\pi$

8. amplitude = 5, period =  $7\pi$

9. amplitude = 4, period =  $8\pi$

10. amplitude =  $\frac{3}{5}$ , period =  $\frac{\pi}{3}$

**Write an equation of the cosine function with each amplitude and period.**

11. amplitude =  $\frac{1}{2}$ , period =  $2\pi$

12. amplitude = 7, period =  $4\pi$

13. amplitude =  $\frac{2}{3}$ , period =  $6\pi$

14. amplitude =  $\frac{1}{4}$ , period =  $\frac{8\pi}{3}$

15. amplitude = 6, period =  $\frac{\pi}{4}$

16. amplitude = 11, period =  $\frac{\pi}{2}$



**State the amplitude and period of each function. Then graph at least one period of the function.**

17.  $y = -3 \sin \theta$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

18.  $y = \cos 2\theta$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

19.  $y = 4 \sin \frac{\theta}{2}$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

20.  $y = -\frac{1}{2} \cos 3\theta$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

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**2.05 Graphing Sine and Cosine- Phase Shift Notes**For  $y = a\sin[b(\theta - c)] + d$  or  $y = a\cos[b(\theta - c)] + d$ Phase Shift- horizontal shift; 'c' tells you the phase shift; direction is opposite of what you think; you must factor out 'b' to see the phase shift!

Examples: Identify the transformations that occur on the parent function and graph the function.

1.  $y = \sin\left(\theta - \frac{3\pi}{4}\right)$

2.  $y = \cos(2\theta + \pi)$

**2.05 Practice: Graphing Sine & Cosine with Amplitude, Period, and Phase Shift****Write an equation of the sine function given the following information.**

1) amp. = 3.5 and a period of  $14\pi$

2) amp. =  $\frac{1}{2}$ , period =  $\frac{1}{3}$ , and phase shift right  $\frac{1}{2}$

**Write an equation of the cosine function given the following information.**

3) amp = 1.25, period= $6\pi$ , & phase shift right  $3\pi$   
down 3

4) amp. =  $\frac{2}{3}$ , period =  $\frac{3}{4}$ , and phase shift

**For each function, state the amplitude and period. Then label the axes appropriately and sketch the graph.**

5)  $y = 3 \cos 4\theta$

6)  $y = -4 \sin \frac{\pi\theta}{2}$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ PS: \_\_\_\_\_

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ PS: \_\_\_\_\_

$$7) y = \frac{5}{3} \cos\left(2\theta + \frac{\pi}{2}\right)$$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ PS: \_\_\_\_\_

$$8) y = 2.5 \sin(\theta - \pi)$$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ PS: \_\_\_\_\_

$$9) y = 6 \cos\left(\frac{\theta}{4} - \frac{\pi}{8}\right)$$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ PS: \_\_\_\_\_

$$10) y = 2 \sin\left(\frac{\theta}{2} + \frac{3\pi}{2}\right)$$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ PS: \_\_\_\_\_

$$11) y = -3 \cos(3\pi\theta + \pi)$$

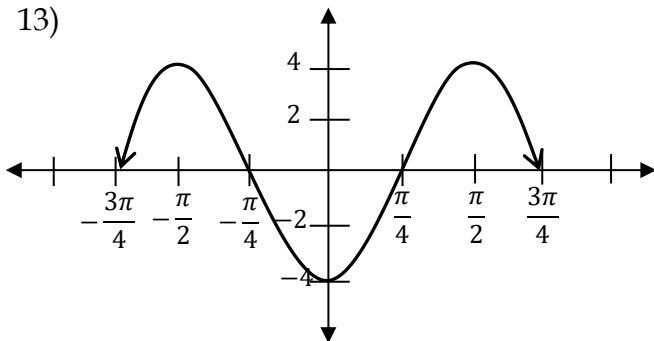
Amp: \_\_\_\_\_ Per: \_\_\_\_\_ PS: \_\_\_\_\_

$$12) y = \sin\left(\frac{\theta}{6} - \frac{\pi}{2}\right)$$

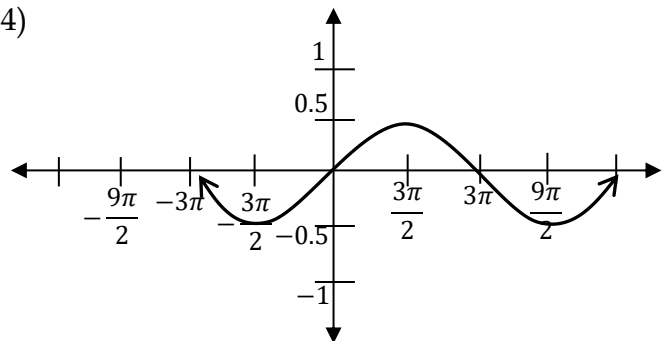
Amp: \_\_\_\_\_ Per: \_\_\_\_\_ PS: \_\_\_\_\_

Write a sine equation and a cosine equation for each graph below:

13)



14)



**Accel Pre-Calculus**

Name: \_\_\_\_\_

**2.06 Graphing Sine and Cosine- Vertical Shift Notes**For  $y = a\sin[b(\theta - c)] + d$  or  $y = a\cos[b(\theta - c)] + d$ 

Vertical Shift- amount graph moves up or down; 'd' tells you the vertical shift; direction is what you assume

Examples: Identify the transformations that occur on the parent function and graph the function.

1.  $y = \sin\left(\theta - \frac{\pi}{4}\right) + 3$

2.  $y = \cos\left(3\theta + \frac{\pi}{2}\right) - 1$

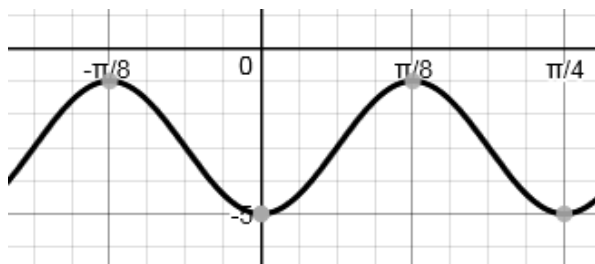
**2.06 Worksheet: Graphing Sine and Cosine with Amplitude, Period, Phase Shift and Vertical Shift**

Date: \_\_\_\_\_

Write the function described:

1. A sine function with amplitude = 15, with a reflection, period =  $4\pi$ , phase shift right  $\frac{\pi}{2}$  and vertical shift down 10.
2. A cosine function having an amplitude =  $\frac{1}{2}$ , period =  $\frac{\pi}{3}$ , phase shift left  $\frac{\pi}{3}$  and vertical shift up 5.
3. A cosine function that has been vertically stretched by a factor of 4, has been reflected over the x-axis, has been horizontally compressed to a period of  $\frac{2\pi}{3}$ , and has been shifted right  $\pi$  units and down 3 units.
4. A sine function that has been horizontally stretched to a period of 10, vertically compressed by a factor of  $\frac{2}{3}$ , shifted left  $\frac{5}{2}$ , and up 1 unit.

5. The wave graphed below. It is **both**: a sine function and a cosine function, with different phase shifts.



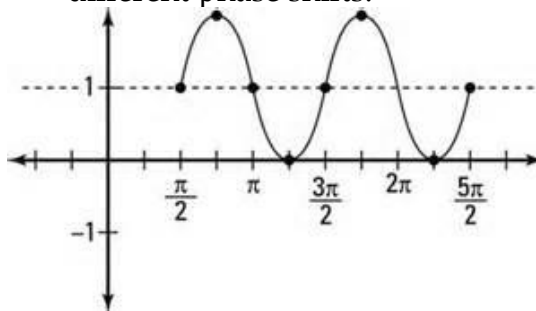
Given  $y = a \sin[b(\theta - c)] + d$  and  $y = a \cos[b(\theta - c)] + d$ . For each function, state the amplitude, period, interval, phase shift (PS), and vertical shift (VS). Sketch the graph and label the axes.

7)  $y = -2 \sin \frac{\theta}{2} + 1$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ Int: \_\_\_\_\_

PS: \_\_\_\_\_ VS: \_\_\_\_\_

6. The wave graphed below. It is **both**: a sine function and a cosine function, with different phase shifts.



8)  $y = 2 \cos \left( \theta + \frac{\pi}{2} \right) - 3$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ Int: \_\_\_\_\_

PS: \_\_\_\_\_ VS: \_\_\_\_\_

9)  $y = \frac{1}{2} \sin \left( \frac{\theta}{6} + \frac{\pi}{3} \right) + 4$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ Int: \_\_\_\_\_

PS: \_\_\_\_\_ VS: \_\_\_\_\_

10)  $y = -\cos (6\pi\theta + \pi) - 1$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ Int: \_\_\_\_\_

PS: \_\_\_\_\_ VS: \_\_\_\_\_

11)  $y = 3 \sin \left( 2\theta + \frac{3\pi}{2} \right) + 0.5$

Amp: \_\_\_\_\_ Per: \_\_\_\_\_ Int: \_\_\_\_\_

PS: \_\_\_\_\_ VS: \_\_\_\_\_

12)  $y = -\cos \left( \frac{\pi}{4}\theta + \frac{\pi}{2} \right) - 1.5$

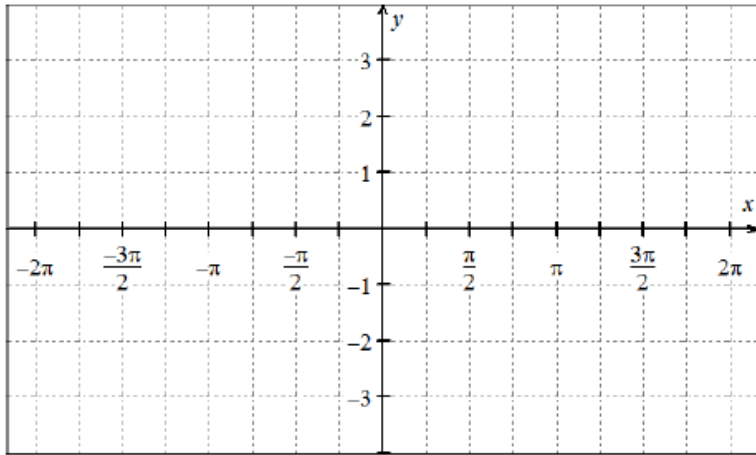
Amp: \_\_\_\_\_ Per: \_\_\_\_\_ Int: \_\_\_\_\_

PS: \_\_\_\_\_ VS: \_\_\_\_\_

Pre-Calculus- More Sine and Cosine Graphing Review

Graph at least TWO periods of the function, state the amplitude, period, phase shift, and vertical shift. Plot the critical points.

$$y = 3 \sin \left( 2\theta - \frac{\pi}{2} \right) - 1$$



Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Intervals: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

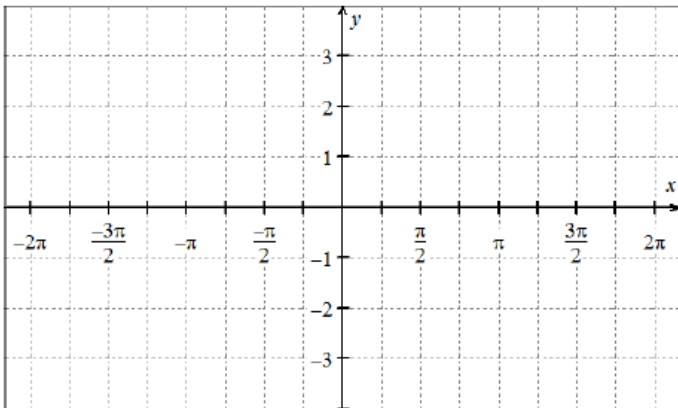
Vertical Shift: \_\_\_\_\_

Write the equation of the cosine function with amplitude 5, period  $7\pi$ , phase shift right  $\frac{\pi}{2}$ , and vertical shift up 3.

$$y = \underline{\hspace{10em}}$$

Graph at least one period of the function, state the amplitude, period, phase shift, and vertical shift. Plot the critical points.

$$y = -2 \cos \left( \frac{2\theta}{3} + \pi \right) + 2$$



Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Intervals: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

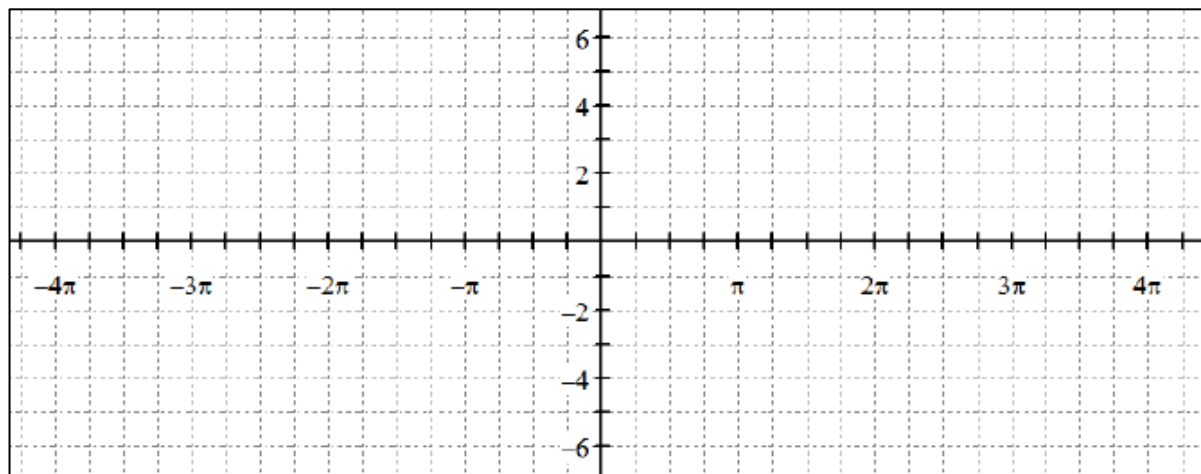
Vertical Shift: \_\_\_\_\_

Write the equation of the sine function with amplitude 2.4, period  $\frac{3\pi}{4}$ , phase shift left  $\frac{\pi}{2}$ , and vertical shift down 6.

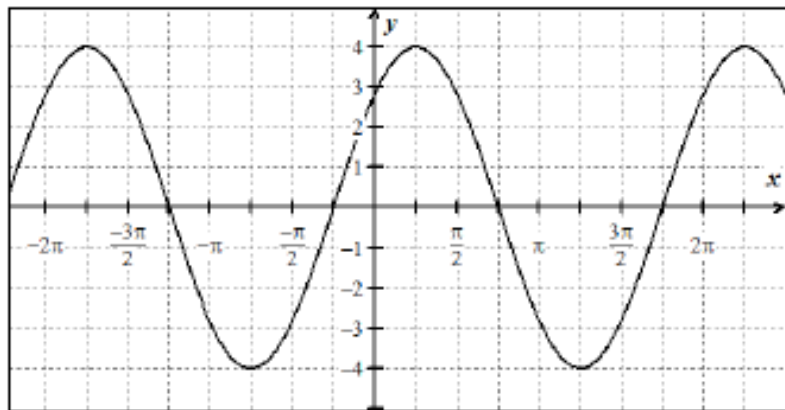
$$y = \underline{\hspace{10em}}$$

Graph at least TWO periods of the function, state the amplitude, period, phase shift, and vertical shift. Plot the critical points.

$$y = 4 \cos\left(\theta + \frac{3\pi}{4}\right) - 2$$



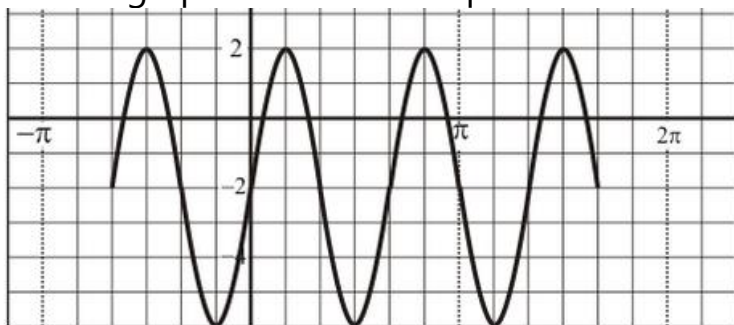
Amplitude: \_\_\_\_\_ Period: \_\_\_\_\_ Intervals: \_\_\_\_\_  
 Phase Shift: \_\_\_\_\_ Vertical Shift: \_\_\_\_\_



Use the graph to write the equation of the sine function.

$y =$  \_\_\_\_\_

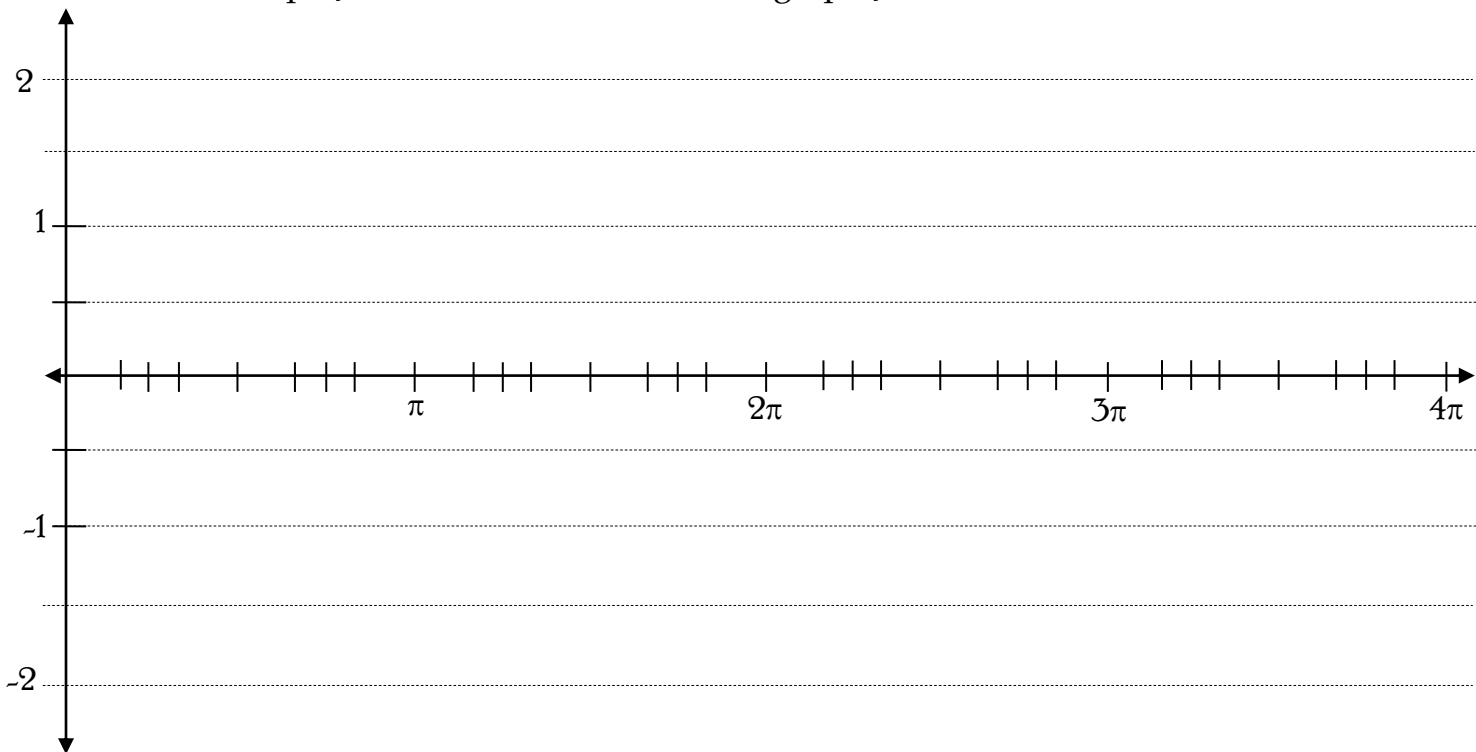
Use the graph to write the equation of the sine function.



$y =$  \_\_\_\_\_

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
Sine (decimal)	0	.5	.707	.866	1	.866	.707	.5	0	-5	-.707	-.866	-1	-.866	-.707	-.5	0
Cosecant (decimal)																	

Graph  $y = \sin \theta$  in one color. Then graph  $y = \csc \theta$  in another color.



The period of  $y = \csc \theta$  is \_\_\_\_\_

The domain of  $y = \csc \theta$  is \_\_\_\_\_

The range of  $y = \csc \theta$  is \_\_\_\_\_

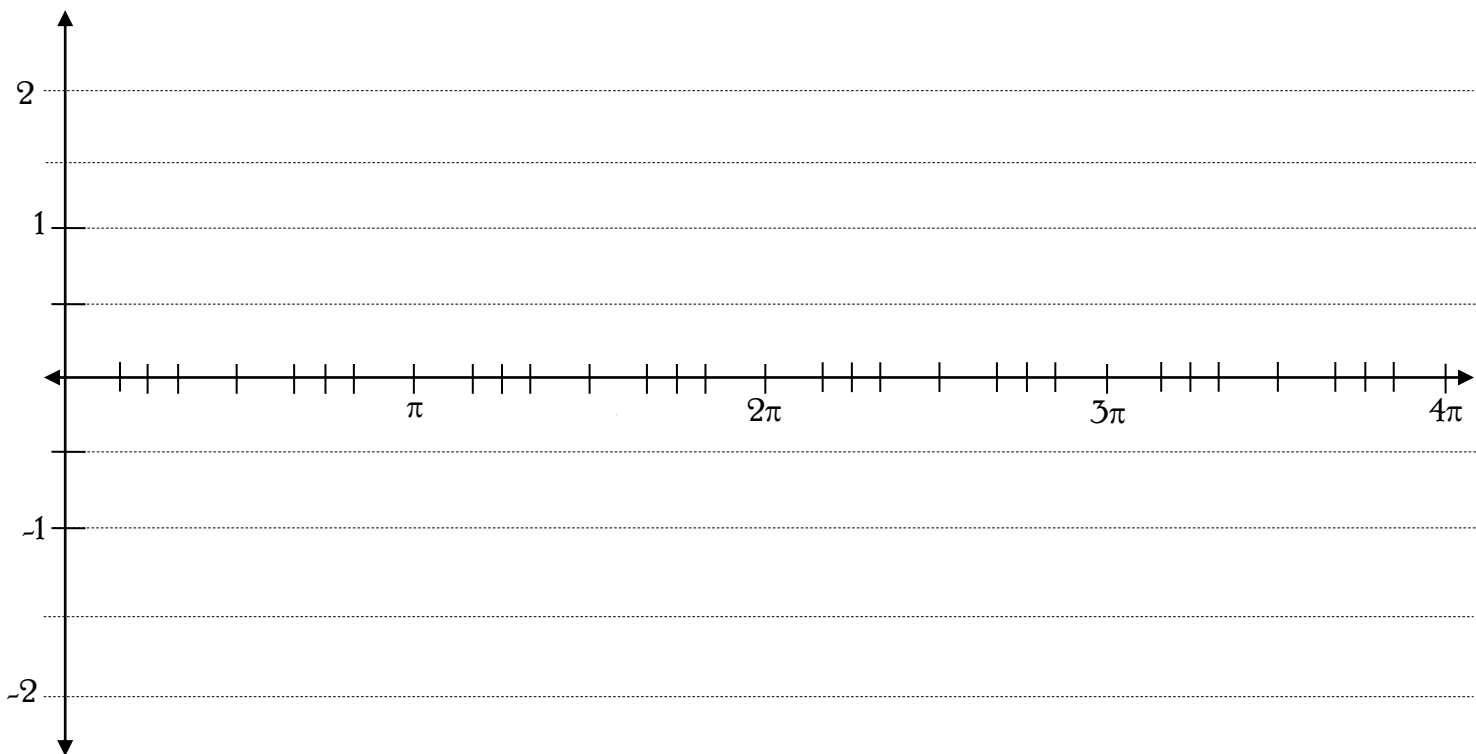
**To graph cosecant functions:**

1. Plot your points like you're graphing \_\_\_\_\_
2. You'll have vertical asymptotes where you see points on the \_\_\_\_\_
3. Plot parabolas at \_\_\_\_\_ and \_\_\_\_\_ points. \_\_\_\_\_ points open up and \_\_\_\_\_ points open down



Now let's do the same for cosine and secant.

Graph  $y = \cos \theta$  in one color. Then graph  $y = \sec \theta$  in another color.



**Graph each function by first graphing their reciprocal trigonometric function.**

1.  $y = -3 \csc\left(\frac{\theta}{4} + \frac{\pi}{8}\right) - 2$

2.  $y = -0.5 \sec(2\theta - \pi) + 3$

Vertical Stretch: \_\_\_\_\_

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

**2.09 Practice: Graphing Secant and Cosecant Functions**

Date \_\_\_\_\_

**Graph each.**

1.  $y = \csc 3\theta$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

2.  $y = -\sec\left(\theta - \frac{\pi}{2}\right)$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

3.  $y = 5 \sec \theta + 3$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

4.  $y = \csc\left(\frac{\theta}{4} + \frac{\pi}{2}\right)$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

5.  $y = 2 \csc(4\theta - \pi) - 2$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

6.  $y = -3 \csc\left(6\theta + \frac{3\pi}{2}\right) + 1$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

$$7. y = \sec\left(\frac{\pi\theta}{5} - \frac{\pi}{2}\right) - 1.5$$

$$8. y = \csc(2\pi\theta + \pi)$$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

**Write an equation for each function described.**

9. Secant function, vertically stretched to be 5 times taller than normal, horizontally compressed to have a period one-half the length as normal, shifted  $\frac{\pi}{4}$  units to the right and 10 units down.

10. Cosecant function, reflected upside down, period =  $\frac{\pi}{3}$ , phase shift right  $\frac{\pi}{2}$ , and vertical shift up 4

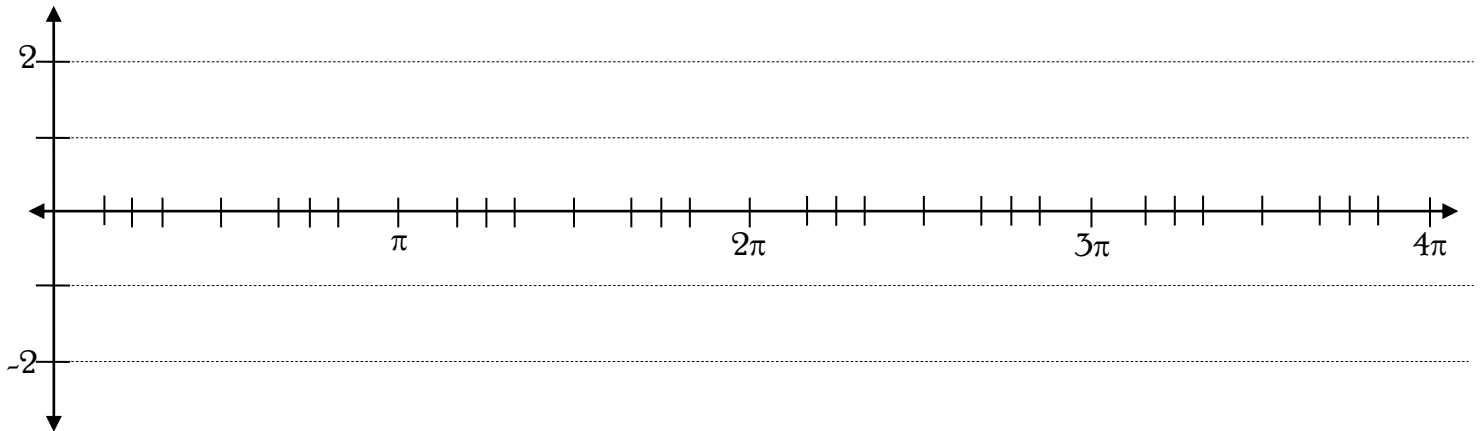
11. Secant function, period = 8, phase shift right 5, and vertical shift up 7.5

## 2.10: Exploring the Cotangent & Tangent Graphs

Date \_\_\_\_\_

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
cot $\theta$ (exact)																	
cot $\theta$ (decimal)																	

Graph  $y = \cot \theta$ .



The period of  $y = \cot \theta$  is \_\_\_\_\_

The domain of  $y = \cot \theta$  is \_\_\_\_\_

The range of  $y = \cot \theta$  is \_\_\_\_\_

1. Graph  $y = 5 \cot \theta - 1$

Vertical Stretch: \_\_\_\_\_

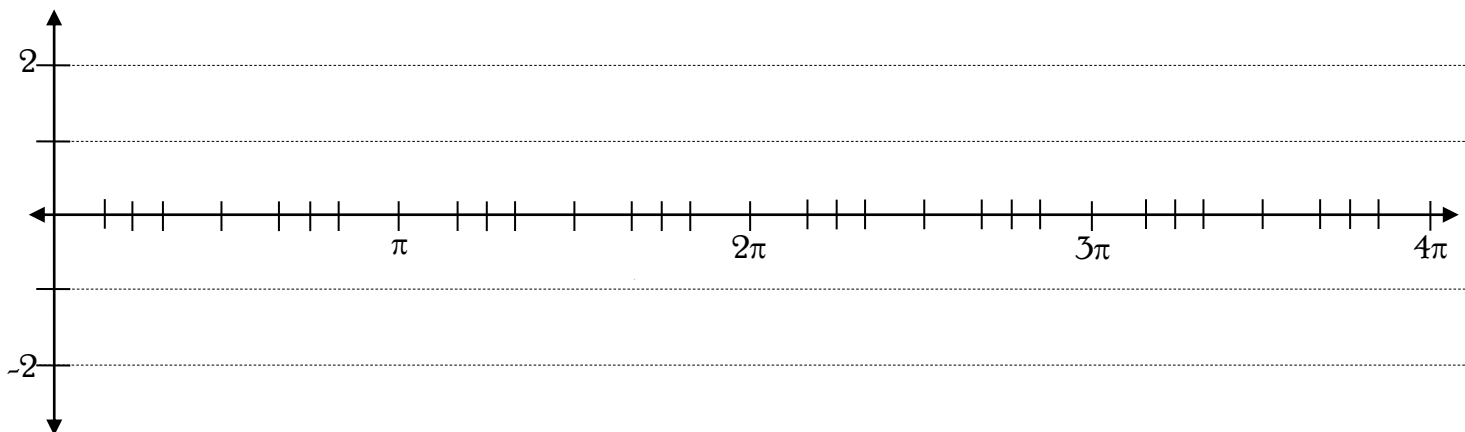
Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
$\tan \theta$ (exact)																	
$\tan \theta$ (decimal)																	

Graph  $y = \tan \theta$



The period of  $y = \tan \theta$  is \_\_\_\_\_

**Practice: Graph one period of tangent.**

2.  $y = \tan\left(\frac{\theta}{2} - \frac{3\pi}{4}\right)$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

3.  $y = -2 \tan\left(3\theta - \frac{\pi}{4}\right) - 1$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

**2.10 Practice: Graphing Tangent and Cotangent Functions** Date \_\_\_\_\_

**Graph each.**

1.  $y = \cot 3\theta$

2.  $y = \tan\left(\theta - \frac{\pi}{2}\right)$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

3.  $y = -\tan \theta - 2$

4.  $y = \cot\left(\frac{\theta}{4} + \frac{\pi}{2}\right)$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

$$5. y = \cot\left(4\theta - \frac{\pi}{2}\right) + 2$$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

$$6. y = -\cot\left(3\theta + \frac{\pi}{2}\right) + 1$$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

$$7. y = 2 \tan\left(\frac{\theta}{2} - \frac{\pi}{2}\right) - 3$$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

$$8. y = -3 \cot(2\theta + 3\pi) + 5.5$$

Per: \_\_\_\_\_ PS: \_\_\_\_\_ VS: \_\_\_\_\_

**Write an equation for each function described.**

9. Tangent function, period =  $3\pi$ , phase shift left  $\frac{\pi}{4}$ , and vertical shift down 6

10. Cotangent function, period =  $\frac{\pi}{4}$ , phase shift right  $\frac{\pi}{2}$ , and vertical shift up 9

## 2.11 Additional Practice: Graphing Sec, Csc, Tan, Cot

Date \_\_\_\_\_

Find the Vertical Stretch, Period, Phase Shift, and Vertical Shift. Then graph the function.

1.  $y = \csc\left(4\theta + \frac{\pi}{2}\right)$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

2.  $y = \sec\left(\frac{\theta}{2} - \pi\right)$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

3.  $y = -3 \sec(2\theta) + 1$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

4.  $y = 2.5 \csc\left(\theta - \frac{\pi}{4}\right) - 5$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_



5.  $y = 2 \tan \theta + 4$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

6.  $y = \cot\left(2\theta - \frac{\pi}{2}\right)$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

7.  $y = -3 \cot\left(\frac{\theta}{3} + \frac{\pi}{6}\right) + 1$

Vertical Stretch: \_\_\_\_\_

Period: \_\_\_\_\_

Phase Shift: \_\_\_\_\_

Vertical Shift: \_\_\_\_\_

**Write the equation described.**

8. A secant function that has been shifted to the left  $\frac{\pi}{3}$  units and down 4 units, and that has a wavelength of  $\pi$  units.

9. A cotangent function that has a period of  $3\pi$ , has been shifted to  $\frac{\pi}{2}$  units to the right, and has been shifted 4 units up.

2.13 Unit 2 Review: Graphing Trig Functions & Sinusoidal Modeling    Date: \_\_\_\_\_

State the requested characteristics of each function. Graph at least one period of the function.

1.  $y = \sin\left(\frac{\theta}{4} + \frac{3\pi}{8}\right) + 1$

Amplitude \_\_\_\_\_

Period \_\_\_\_\_

Phase Shift \_\_\_\_\_

Vertical Shift \_\_\_\_\_

2.  $y = -4 \sec(\theta - \pi) - 2$

Vertical Stretch \_\_\_\_\_

Period \_\_\_\_\_

Phase Shift \_\_\_\_\_

Vertical Shift \_\_\_\_\_

3.  $y = 5 \tan\left(\frac{\theta}{2} + \frac{\pi}{2}\right) + 3$

Vertical Stretch \_\_\_\_\_

Period \_\_\_\_\_

Phase Shift \_\_\_\_\_

Vertical Shift \_\_\_\_\_

4.  $y = 0.5 \cot(2\theta + \pi) - 1$

Vertical Stretch \_\_\_\_\_

Period \_\_\_\_\_

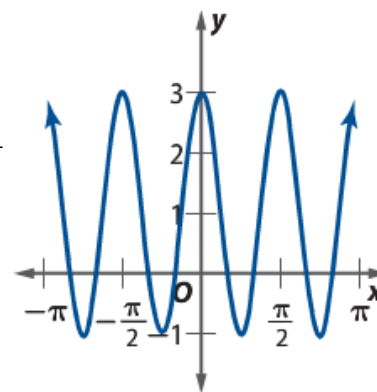
Phase Shift \_\_\_\_\_

Vertical Shift \_\_\_\_\_

**Write two functions for each graph using the specified functions.**

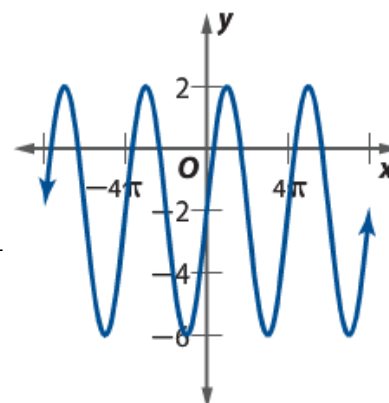
5. Using sine: \_\_\_\_\_

Using cosine: \_\_\_\_\_



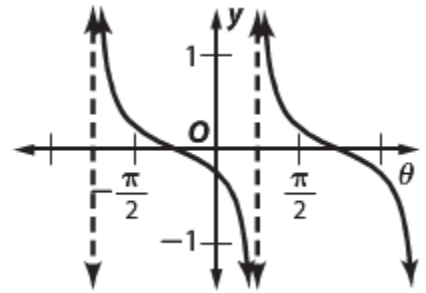
6. Using sine: \_\_\_\_\_

Using cosine: \_\_\_\_\_



7. Using tangent: \_\_\_\_\_

Using cotangent: \_\_\_\_\_



8. True or False, and explain why: Every sine function of the form  $y = a \sin b(x - c) + d$  can also be written as a cosine function of the form  $y = a \cos b(x - c) + d$ .

9. True or False, and explain why: The period of  $f(x) = \cos 8\theta$  is equal to four times the period of  $g(x) = \cos 2\theta$ .

10. True or False, and explain why: If  $x = \theta$  is an asymptote of  $y = \csc x$ , then  $x = \theta$  is also an asymptote of  $y = \cot x$ .

11. How many zeros does  $y = \cos 1500\theta$  have on the interval from  $0 \leq \theta \leq 2\pi$ ?

**2.15 Applications of Sinusoidal Functions Day 1**

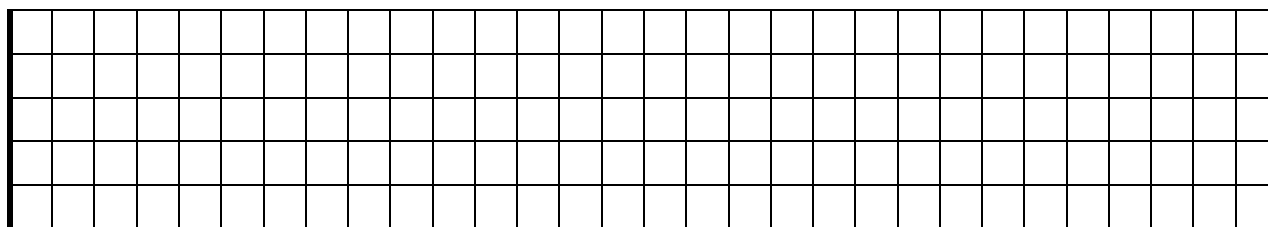
**Date:** \_\_\_\_\_

**Problem 1: Moon Watch**

As an avid sky-watcher, you know that the moon is always half illuminated by the sun and that how much of the moon we can see illuminated depends on where it is in its orbit around Earth. For the first months of the year, you watched the moon and kept data for the amount of the moon that was visible each day.

Day of Year	1	4	7	9	13	17	20	24	27	30	35	38	43	47	50	52	57	59
Proportion of Moon Visible	0.5	0.81	0.98	1	0.82	0.46	0.17	0	0.16	0.5	0.92	1	0.79	0.41	0.12	0	0.33	0.5

**Part 1. Graph this relationship.** (Hint: scale the x-axis by 2)



**Part 2. a) Explain the practical meaning of each of the following characteristics.**  
**b) Find the value of each characteristic for this application.**

Sinusoidal Function:

Period:

Vertical Shift:

Amplitude:

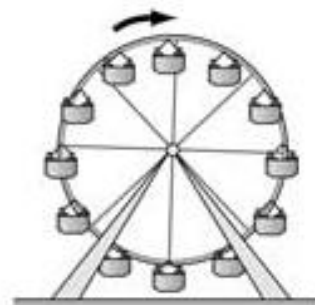
Phase Shift:

**Part 3. Write the equation of the sinusoidal function that models the visibility of the moon.**

### Problem 2: A Day at the Amusement Park

There are many rides at the amusement park whose movement can be modeled using trigonometric functions. The Ferris wheel is a good example of periodic movement.

You want to ride the Ferris wheel. It has a radius of 95 feet and is suspended 20 feet above the ground. The wheel rotates at a rate of 2 revolutions every 20 minutes.



#### Part 1.

- What is the period of the function that models the movement of the Ferris wheel?
- What is midline of the function? What is the practical meaning of midline?
- What is amplitude of the function? What is the practical meaning of amplitude?
- Write the equation of a sinusoidal function that models your height above the ground over time.

e) Graph the function:



#### Part 2.

- Determine your height above the ground at 6 minutes.
- Suppose the Ferris wheel moved faster and completed a revolution in 5 minutes. How would the function change? (Rewrite the equation reflecting this change).
- If the radius of the Ferris wheel remained the same, but the height of the wheel was raised 15 feet higher, how would the function change? (Rewrite the equation reflecting this change).

### Homework: Global Warming?

Scientists are continually monitoring the average temperatures across the globe to determine if Earth is experiencing climate change. One statistic scientists use to describe the climate of an area is average monthly temperature. The average monthly temperature of a region is the mean of its average high and low temperatures.

The table below shows the average monthly temperature ( $^{\circ}\text{F}$ ) in Atlanta from January to December.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
40 $^{\circ}$	44 $^{\circ}$	53 $^{\circ}$	61 $^{\circ}$	68 $^{\circ}$	76 $^{\circ}$	80 $^{\circ}$	78 $^{\circ}$	72 $^{\circ}$	62 $^{\circ}$	53 $^{\circ}$	45 $^{\circ}$

**Part 1. Graph this relationship.** Use January as month 1 of the year, meaning that for January,  $t = 1$ .


### Part 2.

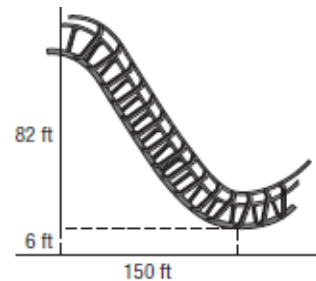
- What is the period of the function that models the monthly temperatures in Atlanta? Find  $b$ .
- What is the vertical shift of this model? Find  $k$ .
- What is the amplitude of this model? Find  $a$ .
- What is the phase shift of this model? Find  $c$ .
- Write the equation of a sinusoidal function that models the average monthly temperatures.
- Use the equation to predict the temperature in October.

## 2.16 Sinusoidal Modeling Day 2

Date \_\_\_\_\_

1. A certain person's blood pressure oscillates between 140 and 80. If the heart beats once every second, write a sine function that models the person's blood pressure.
2. A rodeo performer spins a lasso in a circle perpendicular to the ground. The height of the knot from the ground is modeled by  $h = -3 \cos\left(\frac{5\pi}{3}t\right) + 3.5$ , where  $t$  is the time measured in seconds.
  - a. What is the highest point reached by the knot?
  - b. What is the lowest point reached by the knot?
  - c. What is the period of the model?
  - d. According to the model, find the height of the knot after 25 seconds.

3. Part of a roller coaster track is a sinusoidal function. The high and low points are separated by 150 feet horizontally and 82 feet vertically as shown. The low point is 6 feet above the ground.
  - a. Write a sinusoidal function that models the distance the roller coaster track is above the ground at a given horizontal distance  $x$ .



- b. Point A is 40 feet to the right of the  $y$ -axis. How far above the ground is the track at point A?
4. A buoy, bobbing up and down in the water as waves pass it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 3 feet.
    - a. Determine the amplitude and period of a sinusoidal function that models the bobbing buoy.
    - b. Write an equation of a sinusoidal function that models the bobbing buoy, using  $x = 0$  as its highest point.



5. The average monthly temperatures for Baltimore, Maryland, are shown below.

- a. Determine the amplitude, period, phase shift, and vertical shift of a sinusoidal function that models the monthly temperatures using  $x = 1$  to represent January.

Month	Temperature (°F)	Month	Temperature (°F)
Jan	32	July	77
Feb	35	Aug	76
Mar	44	Sept	69
Apr	53	Oct	57
May	63	Nov	47
June	73	Dec	37

- b. Write an equation of a sinusoidal function that models the monthly temperatures.

- c. According to your model, what is Baltimore's average temperature in July? In December?

6. One day all 322 million people in the United States climb up on tables. We all jump off and land on the floor/ground at  $t = 0$ . The resulting shock as we hit Earth's surface will start the entire earth vibrating in such a way that its surface first moves down from its normal position. At  $t = 300$  milliseconds, it has moved up an equal distance above its normal position. The total difference in Earth's vertical position is 6 mm. For an extremely brief amount of time, the vertical position of the surface closely resembles a sinusoidal function.

- a. Sketch a graph of Earth's vertical movement over time for this scenario.

b. At what time will the first minimum occur?

c. What is the period of this function?

d. What is the amplitude of this function?

e. Write an equation of a sinusoidal function that models Earth's vertical position.

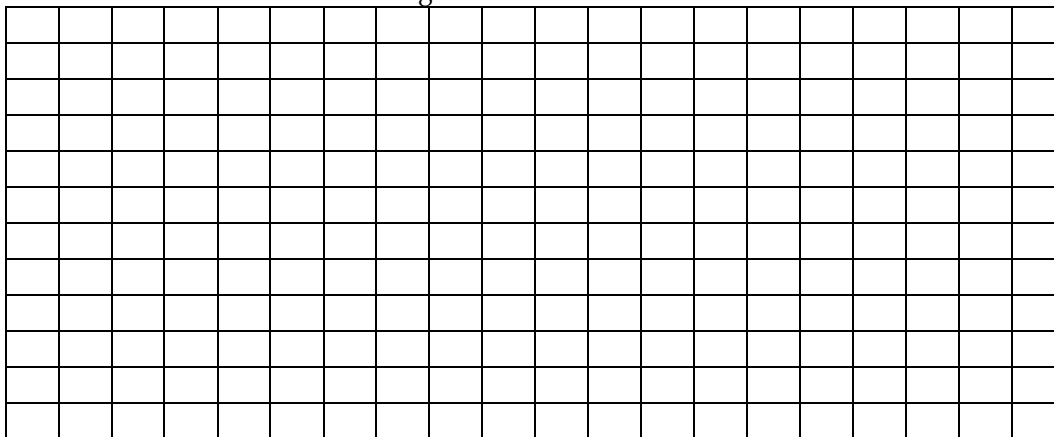
f. If the wave were to continue, predict the vertical position of Earth 3.25 seconds after our jump.

2.17 Sinusoidal Modeling Day 3

Date: \_\_\_\_\_

1. In Canada's wonderland there is a roller coaster that is a continuous series of identical hills that are 18m high from the ground. The platform to get on the ride is on top of the first hill. It takes 3 seconds for the coaster to reach the bottom of the hill 2m off the ground.

a) Use the information above to sketch a diagram of this sinusoidal movement.



b) Write a cosine function,  $h(t)$ , that describes the situation in part a.

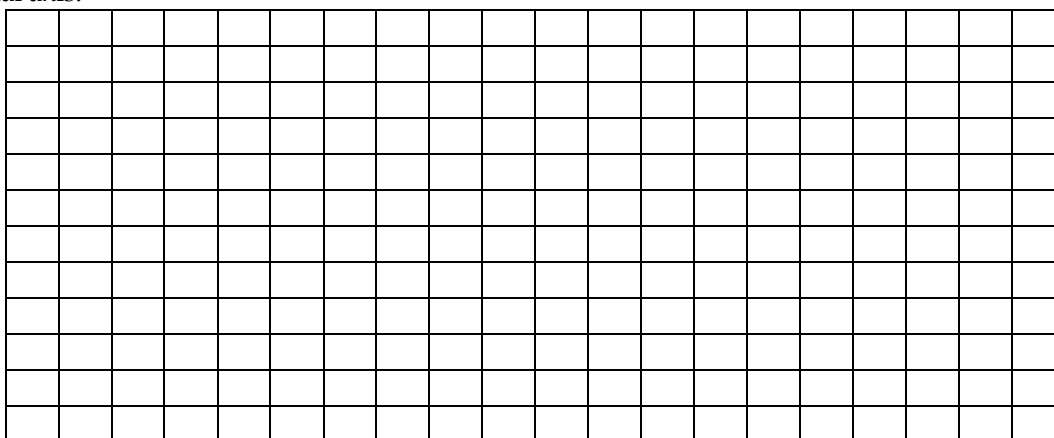
b) \_\_\_\_\_

c) Determine the height of the rider at 11 seconds. Answer in function notation.

c) \_\_\_\_\_

2. Mr. Jones, disguised as Mathman, a costumed crime fighter, is swinging back and forth in front of the window for the Front Office. At  $t = 3$  sec, he is at one end of his swing and 4m from the window. At  $t = 7$  sec, he is at the other end of his swing and 20m from the window.

a) Sketch the curve. Use the distance from the window on the vertical axis and the time in seconds along the horizontal axis.



b) Write a sine function that describes the situation in part a.

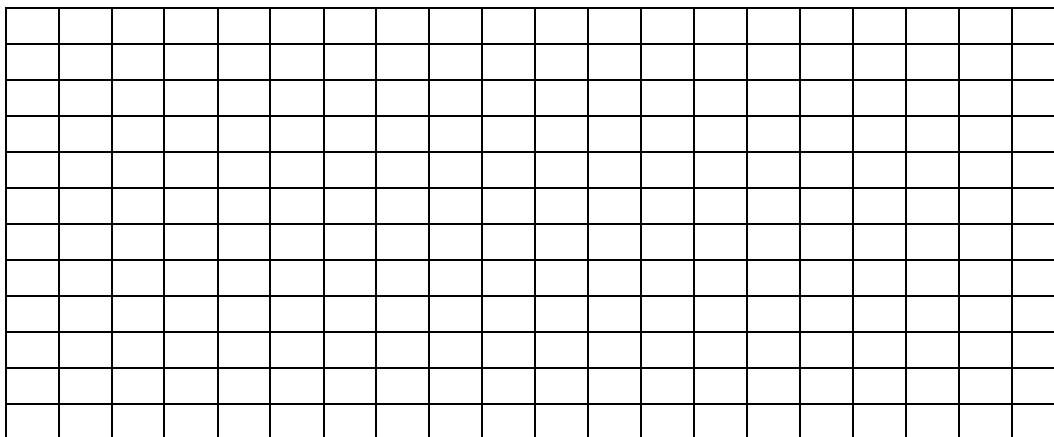
b) \_\_\_\_\_

c) When is the first time Mathman reaches 16m?

c) \_\_\_\_\_

3. John is floating on a tube in a wave tank. At  $t = 1$  second, John reaches a maximum height of 14m above the bottom of the pool. At  $t = 9$  seconds, John reaches a minimum height of 2m above the bottom of the pool

a) Use the information above to sketch a diagram of this sinusoidal movement.



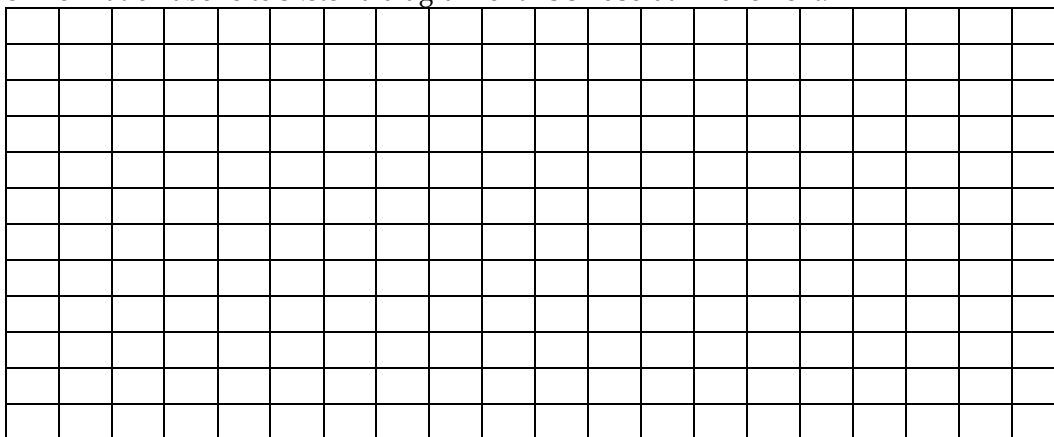
b) Write a cosine function that describes the situation in part a.

b) \_\_\_\_\_

c) What is John's height from the bottom of the pool at 21 seconds? c) \_\_\_\_\_

4. A pendulum on a grandfather clock is swinging back and forth as it keeps time. A device is measuring the distance the pendulum is above the floor as it swings back and forth. At the beginning of the measurements the pendulum is at its highest point, 36 cm high exactly one second later it was at its lowest point of 12 cm. One second later it was back to its highest position.

a) Use the information above to sketch a diagram of this sinusoidal movement.



b) Write a cosine function that describes the situation in part a.

b) \_\_\_\_\_

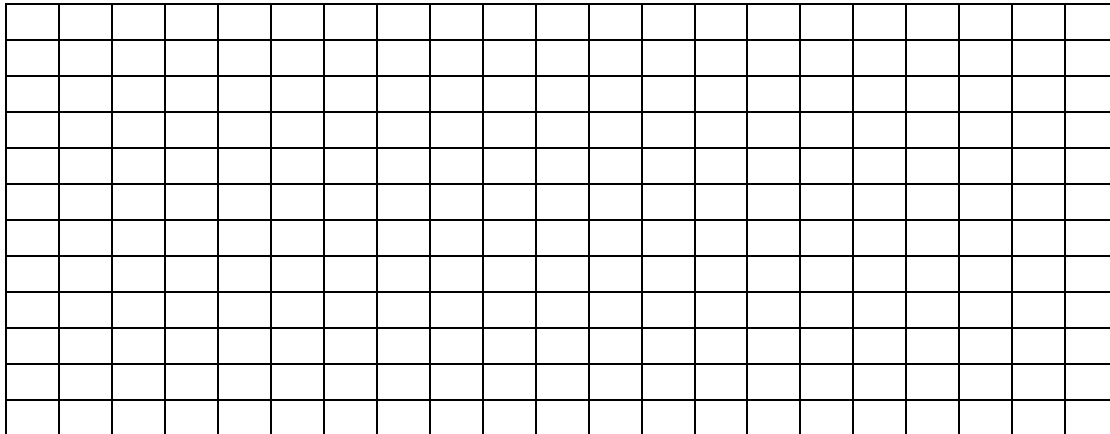
c) Write a sine function that describes the situation in part a.

c) \_\_\_\_\_

d) When will the pendulum be at 15 cm if  $2 < t < 3$ ?

d) \_\_\_\_\_

5. Sam is riding his bike one day and picks up a nail in his tire. The nail hits the ground every 2 seconds and reaches a maximum height of 48 cm (assume the tire does not deflate).
- a) Use the information above to sketch a diagram of this sinusoidal movement.



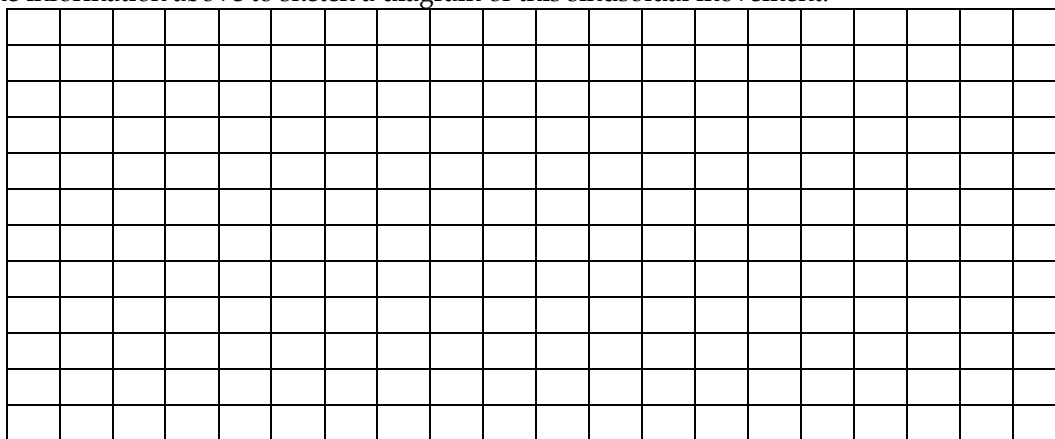
b) Write a cosine function that describes the situation in part a.

b) \_\_\_\_\_

c) Write a sine function that describes the situation in part a.

c) \_\_\_\_\_

6. Jackie, Nicolle and Maegan are playing skip rope. As the rope rotates it is observed that its maximum height is 2.75m after 1 second. The first minimum height of 0.25m occurs 2 seconds after the maximum height.
- a) Use the information above to sketch a diagram of this sinusoidal movement.



b) Write a cosine function that describes the situation in part a.

b) \_\_\_\_\_

c) Write a sine function that describes the situation in part a.

c) \_\_\_\_\_

d) What is the height of the rope at 2.75 seconds?

d) \_\_\_\_\_

7. A Ferris wheel 60 ft in diameter makes one revolution every 50 seconds. If the center of the wheel is 35 feet above the ground, how long after reaching the low point is a rider 50 ft. above the ground? Write the function and the time. Show all your work.

7. Function: \_\_\_\_\_

Time: \_\_\_\_\_

8. Ebb and Flow: on a particular Labor Day, the high tide in South California occurs at 7:15 am. At that time, you measure the water at the end of the Santa Monica Pier to be 12 feet deep. At 1:36 pm, it is low tide and you measure the water to be only 8 feet deep. What is the depth at noon? Write the function and the depth.

8. Function: \_\_\_\_\_

Depth: \_\_\_\_\_